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Energy-Nonconserving Planck Fluctuations and Strong Forces

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On the basis of the recent demonstration that sources of the dual Riemann tensor are violations of local energy-momentum conservation, it is hypothesized that matter is subject to the Heisenberg uncertainty principle because of stochastic Planck-scale fluctuations in the Euclidean geometry of the vacuum. The identification of such singularities with unitons, whose masses are quantized in discrete units of the Planck mass, and also with the sources of "strong gravity," is shown to lead to the correct strength, range, and duration of strong interactions. A vacuum-induced cosmological term, due to coupling of spin to space-time torsion, results in massive gravitons, with mass similar to the spin-2 mesons, and a Yukawa, rather than Newtonian, variation of the hadron gravitational potential, thus adding support to "strong gravity" theories of the strong force.

1. INTRODUCTION

In recent years, the search for unification of nature's known forces in a theory involving a single basic interaction and a unique coupling constant of a spontaneously broken fundamental gauge symmetry group has stimulated interest in the primary role gravitation may play in determining the absolute strengths of the other forces, in particular, the strong interaction, to which attention will be confined here. In "strong gravity" ($f - g$) theories (Isham et al., 1971a) the essential idea is that, in analogy to the vector meson dominance of the electromagnetic coupling of hadrons (mixing of the photon with the ρ, ω, and ϕ mesons), the gravitational interaction of hadrons proceeds only indirectly through the exchange of gravitons (unlike that of the leptons), the graviton mixing with massive spin-2 mesons. It is easily shown (Ross,

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1972) by solving Einstein's field equations with the energy-momentum tensor of a Yukawa field as source that the metric tensor has a Yukawa potential as well as a Newtonian potential present. The former is generated by the "strong" charges carried by hadrons and dominates the latter at distances of the order of magnitude of the hadron Compton wavelength. Leptons, which carry no "strong" charges, generate only the weak "Einstein" gravitational field and so do not interact strongly, their fields being due to their energy alone. Identification is then made of the exchanged spin-2 meson with the members of the spin-2^{+++} nonet with the quantum numbers ($I = Y = 0$) of the vacuum, in particular, the *f* meson (1254 MeV). However, a gravitational explanation of strong interactions should not depend on the existence of such mesons, which, after all, occur in nature only *because* of the strong forces. The essential features that distinguish them from the other forces—strength, range, and duration—should have an explanation that is independent of the energy, distance, and time scale which they set. The latter should, indeed, be derivable, not presupposed as in "strong gravity" theories, which are hence still phenomenological as traditional field theory is, to some extent. In such theories, too, the basic distinction between leptons and hadrons is not explained but, instead, merely upheld and so as fundamental explanations somewhat beg the question.

In the following are offered arguments, based on the notion of an energy-nonconserving vacuum and "dual" gravitational charge, which not only set, a priori, the scale of strong forces but also lend support to the hypothesis of a stochastic basis for quantum mechanics arising from random fluctuations at the Planck level in the gravitational field of the vacuum. Insofar as Einstein's field equations are *not* the equations relating expectation values of the corresponding field operators, the marriage of general relativity to quantum mechanics should not have even been agreed to, let alone consummated. The order-of-magnitude calculations presented here should be seen as pointing towards the underlying physics only and should not be regarded as rigorous in themselves. They serve only to emphasize the possibility of a gravitational origin of strong interactions.

2. "DUAL" GRAVITATIONAL CHARGE AND ENERGY NONCONSERVATION

In an initial step towards a properly quantized theory of gravitation, Motz (1972) has argued that the concept of gravitational charge $G^{1/2}m$ is more relevant for quantum matter than the classical notion of inertial mass, m. Using the method due to Schwinger (1966, 1968) he has shown that the demand for quantization in integer multiples of $\hbar$ (Planck's reduced constant) of the orbital angular momentum of two point particles of mass M, bound by gravity, is equivalent to the following quantization condition on their masses (or rather their gravitational charge):

$$(G^{1/2}M)^2 = n\hbar c \qquad (n = 0, 1, 2, \ldots) \tag{2.1}$$

or

$$M = n^{1/2}M_0 \tag{2.2}$$

where $M_0 = (\hbar c/G)^{1/2}$ is the Planck mass ($M_0 = 2.2 \times 10^{-5}$ g $= 1.22 \times 10^{19}$ GeV). Such superheavy particles or "unitons" may exist in nature. Their identification even with quarks has been suggested (Sivaram and Sinha, 1974). We propose that they exist as spin-1 bosons in uniton–antiuniton pairs (U–$\bar{U}$) into which the graviton can dissociate, just as photons can behave as virtual electron–positron pairs. Whether they exist as free particles, albeit with a very short lifetime, is irrelevant to their role, proposed here, as Goldstone bosons arising from the spontaneous breakdown of Poincaré symmetry due to nonconservation of the vacuum energy. That massive particles are involved somehow in the spontaneous breakdown of unified gauge group symmetry and may lead to the observed disparity in strength of the strong, weak, and electromagnetic interactions has been demonstrated recently (Georgi et al., 1974). Gravitation offers the possibility of a natural mechanism whereby superstrong symmetry breaking of a fundamental gauge group symmetry leads to the rescaling of particle masses from the Planck magnitude down to the level observed in nature.

Relevant to this idea is the demonstration by Isham et al. (1971b) that the Planck mass and the Planck length $L_0 = (\hbar G/c^3)^{1/2}$ are the effective cutoff parameters in the gravitational suppression of ultraviolet infinities in quantum electrodynamics. Notice that the Planck length is just half the Schwarzschild radius of the uniton, indicating the possible role of the uniton as a black hole of the *quantized* gravitational field in providing the first link in the chain of symmetry-breaking mechanisms for which gravity is responsible.

We shall see in Section 3 that it is possible to understand quantum mechanics itself as a statistical description of matter coupled to a metric tensor of a vacuum that undergoes stochastic fluctuations due to the vacuum polarization of the graviton into uniton–antiuniton pairs. That unitons should indeed be regarded as black holes can be seen from the following argument: The radius of the event horizon of the Kerr–Neumann metric generated by a rotating, electrically charged point mass is

$$r_\pm = m \pm (m^2 - a^2 - Q^2)^{1/2} \tag{2.3}$$

where $m = GM/c^2$, $a = J/M_0$ and $Q^2 = Ge^2/c^4$.

Assuming the spin J is quantized

$$J = n\hbar \qquad (n = 0, \tfrac{1}{2}, 1, \ldots)$$

so since $r_\pm$ is real,

$$m^2 - a^2 - Q^2 \geqslant 0$$

or

$$M^4 - \frac{e^2}{G} M^2 - \left(\frac{n\hbar G}{G}\right)^2 \geqslant 0 \tag{2.4}$$

The inequality holds provided $(e^2/G)^2 < -(n\hbar c/G)^2$, which is false. So only the equality is valid and

$$M^2 = \frac{e^2}{2G}\left[1 + \frac{2n}{\alpha}\left(1 + \frac{\alpha^2}{4n^2}\right)^{1/2}\right]$$

and

$$M \simeq n^{1/2} M_0 \left[1 + \frac{\alpha^2}{16n^2} + O\left(\frac{\alpha^4}{n^4}\right)\right] \qquad (n \neq 0) \tag{2.5}$$

$\alpha = e^2/\hbar c$ is the fine-structure constant. Rotating charged black holes thus lie on a Regge-type trajectory which is asymptotically linear and of slope $dJ/dM^2 = G/c^5 = 4.3 \times 10^{-63}$ sec GeV^{-1} $= 1.0(\hbar/M_0{}^2)$. A rotating Kerr ($Q = 0$) black hole with a unique event horizon has mass given by (2.3):

$$M = n^{1/2} M_0$$

the same as (2.2), and Schwarzschild radius $r_S = 2GM/c^2 = 2n^{1/2} L_0$ and Compton wavelength $\lambda = L_0/n^{1/2}$ (these would be equal for a spin-$\frac{1}{2}$ particle). So for intervals of time of the order of magnitude of the Planck time $T_0 = L_0/c = (\hbar G/c^5)^{1/2} = 5.4 \times 10$ sec, the graviton can behave as a massive U–$\bar{U}$ pair of mass $\sim M_0$. This is consistent with Heisenberg's uncertainty principle because

$$M_0 c^2 T_0 = \hbar \tag{2.6}$$

The energy of matter coupled to the huge gravitational fields briefly generated by unitons will fluctuate violently leading to a purely quantum local nonconservation of energy of matter. Now Riegert (1976) has shown that the source of the "dual" Riemann tensor is quantized and pointlike and responsible for a quantized violation of local energy conservation. The reaction 3-force acting on matter at such isolated points is

$$F_r = \tfrac{3}{2}(c^4/G)n \qquad (n = 0, 1, 2, \ldots) \tag{2.7}$$

Note that it is *independent* of $\hbar$ and particle mass and can be repulsive or attractive, an effective source or sink of energy. Now, to first order in r^{-1}, the gravitational force between two unitons separated by a distance r is

$$F = \frac{GM^2}{r^2} = \frac{GM_0{}^2}{r^2} n$$

So if $F \sim F_r$ then $r \sim L_0$. Accordingly, the sources of the "dual" Riemann tensor can be identified with pointlike gravitational charge, i.e., unitons, and gravitational vacuum polarization is the process leading to local energy nonconservation. We see from (2.7) above and (5.11a) of Riegert's paper that $n = 0$ corresponds to local energy conservation and zero divergence of the particle energy-momentum tensor, i.e., to Einstein's general theory of relativity. It also means uncharged unitons are massless whilst charged unitons have mass given by (using 2.4)

$$M = e/G^{1/2} = 1.0 \times 10^{18}\,\text{GeV} \tag{2.8}$$

Thus $n = 0$ corresponds to the classical long-range gravitational field whilst $n \neq 0$ can be interpreted as corresponding to a massive graviton, which, interacting with matter, results in the exchange of energy between matter and the vacuum and (as we shall see) the breakdown of classical mechanics and Einstein's relativity, in which the vacuum plays no dynamical part. The position of geometrical points in the space-time continuum consequently become imprecise to an extent that is easily estimated: The uncertainty

$$\Delta P = F\Delta t$$

in momentum is, by (2.7),

$$\Delta P \sim \tfrac{3}{2}(c^4/G)\Delta t \tag{2.9}$$

The uncertainty in position is $\Delta x = c\Delta t$ and so the uncertainty principle

$$\Delta P \Delta x \geqslant \hbar$$

gives $\Delta x \geqslant \frac{2}{3}^{1/2}L_0 = 4.4 \times 10^{-44}$ sec. This is also the order of magnitude of the separation of the uniton–antiuniton pair. For consider the massive graviton as an U–$\overline{U}$ pair bound in quantized Bohr orbitals. The radii are given by

$$r_n = \tfrac{1}{2}(3n)^{1/2}L_0 \tag{2.10}$$

and the gravitational binding energy is

$$-B = \frac{GM^2}{2r_n} = \left(\frac{n}{3}\right)^{1/2} M_0c^2 \tag{2.11}$$

The rest mass energy of the bound state is

$$M'c^2 = 2(P_n{}^2c^2 + M^2c^4)^{1/2} + B = (3n)^{1/2}M_0c^2 \tag{2.12}$$

so that the massive graviton has Compton wavelength $\lambda_C = \hbar/M'c = L_0/(3n)^{1/2}$, comparing with the uniton Compton wavelength of $L_0/n^{1/2}$. A space-time description in terms of dispersion-free variables breaks down at the Planck level. If we take the geometrodynamical view that the ultimate elements of matter (quarks or partons) are space-time "bubbles" of radius of curvature $\sim L_0$, propagating like phonons in a nonrigid crystal lattice, then energy nonconservation of quantum particles corresponds to inelastic Umklapp-type interactions with the vacuum in which discrete, quantized amounts of energy and momentum are transferred between the particle and the vacuum (lattice) as a whole, with L_0^{-1} being analogous to the reciprocal lattice vector in terms of which phonon momenta are measured. Motz's formula can now be seen in a new light. Instead of expressing the quantization of the inertial mass of ordinary matter, the source of the Riemann curvature tensor, in units of the Planck mass, it provides the units of "dual" matter, the source of the dual Riemann tensor, which may or may not exist in the free state but which is created in the violent fluctuations of gravitational fields that take place on the Planck space-time scale.

3. VACUUM GRAVITY FLUCTUATIONS AND QUANTUM MECHANICS

"Dual" matter, that is, matter carrying gauge charges dual to the familiar charges of ordinary matter, does not seem to exist in nature, at least as free particles. The magnetic monopole, source of the electromagnetic field dual to that of the electric monopole and generator of the one-dimensional Abelian gauge group $U(I)$, does not seem to occur, except perhaps in dyons bound superstrongly to other monopoles in hadrons (Schwinger, 1969). The non-Abelian electric monopole, dual to the Van't Hoft-type magnetic monopole of non-Abelian gauge symmetry groups, also cannot exist, it may be shown.[2] So "dual" matter, in general, may be relevant only in the physics of the vacuum. Now according to the "Dirac veto" (Wentzel, 1966), the wave function of a Dirac electric monopole vanishes at points simultaneously occupied by a Dirac particle with magnetic charge. They cannot coexist at the same space-time point. Riegert has generalized this result for gravitation by showing that the Dirac field vanishes identically on the world-lines (D) of "dual" matter (Riegert, 1976). These are defined as the sources of the gravitational field described by the dual of the Riemann curvature tensor. Now let us take this result to its logical conclusion. The Heisenberg uncertainty principle forbids the possibility of sharp localization in space-time of an electron with known momentum and energy. In general, such an electron has a nonzero probability of being located at any point in configuration space. But, by the Dirac veto, it cannot exist at the continuous set of points defining the world-line of "dual" matter. So there is no uncertainty that it exists at these points.

[2] Unpublished. Strictly speaking, they cannot coexist with non-Abelian magnetic monopoles. They may, however, be created in pairs in the vacuum polarization of quantized non-Abelian gauge fields.

Now the gravitational field of the electron could become singular there. This would lead to nodes along D in the electron field. But in fact the set of orthonormal tetrads defining the local Lorentz frame and the metric tensor itself both vanish along D! The world-line D is a null curve along which the classical notions of space-time continuum geometry and the principle of equivalence fail. The appearance of a continuous string of zeros in the gravitational field of a Dirac particle must be a purely quantum phenomenon. Of course, since probe electrons would avoid these points, such a breakdown of classical relativity would probably not be directly observable. We certainly cannot, however, avoid the problem by suggesting that the nodal curves are strings of unphysical coordinate singularities similar to the pseudo-Schwarzschild singularity. The coordinate frame used is unspecified and the treatment is quite general. We are forced to admit, therefore, that long D gravitational forces of a noninertial kind originate. These are quantized in magnitude according to (2.7). Since matter fields vanish at their source points, they can only arise from the quantum properties of the vacuum itself. As we shall see later, the order of magnitude of the forces is identical to that acting on matter at the Planck density $\rho_0 = M_0/L_0{}^3 = c^5/\hbar G^2 = 5.2 \times 10^{93}$ g/cm^3. Now the mass density of U–$\bar{U}$ bound states is, by (2.12),

$$\rho \sim M/\lambda_c{}^3 \sim n^2\rho_0 \tag{3.1}$$

and so is comparable to the Planck density. Thus we may see these forces as truly quantum gravitational forces arising from fluctuations in the energy of the vacuum, on the Planck energy-space-time scale, which occur not only cosmologically at the birth of the universe and in the core of black holes but now also in "empty space" itself; that is, gravitational collapse occurs universally and perpetually. The quantum vacuum is a reservoir of energy and between it and matter, energy transactions take place on the Planck scale. Considerations such as these lead to the possibility of understanding quantum mechanics as the necessary description of stochastic interactions of classical particles with a neoclassical vacuum. Instead of seeing (2.7) as a rate of production of 3-momentum of a quantum field that happens not to conserve its energy-momentum, we may alternatively interpret it as a classical (to the extent $\hbar$ is not involved) stochastic force that introduces indeterminism to the classical world. The nonconservation of energy that is allowed in vacuum polarization is not merely because of the uncertainty principle. Such fluctuations in the geometry of the space-time continuum may in fact be responsible for the latter. In this context, it is interesting to note that Nickerson, using a principle originally due to Leibnitz, namely, that matter determines geometry, has argued that the Euclidean geometry of empty space itself is generated by a vacuum of high negative energy density, i.e., that flat space is not empty but rather a sea of fermions of finite (though huge) energies (Nickerson, 1975).

Accordingly, the Einstein field equations are to be modified by the presence of a cosmological term:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa\Sigma_{\mu\nu} \tag{3.2}$$

and the energy-momentum stress tensor $\Sigma_{\mu\nu}$ has a vacuum as well as a matter contribution

$$\Sigma_{\mu\nu} = \Sigma_{\mu\nu}^{vac} + T_{\mu\nu} \tag{3.3}$$

Nickerson then shows that the prescription

$$\Lambda\eta_{\mu\nu} = -\kappa\Sigma_{\mu\nu}^{vac} \tag{3.4}$$

(based on the Leibnitz principle) together with the assumption $T_{\mu\nu} \ll \Sigma_{\mu\nu}^{vac}$ (matter is a weak perturbation of the vacuum energy and geometry) leads to wave-mechanical behavior for a particle of mass m provided

$$\Lambda = aL_0^{-2} \qquad (a > 0, \sim 1, \text{dimensionless}) \tag{3.5}$$

and $mc/\hbar \ll \Lambda^{1/2}$ or $m \ll M_0 \sim 10^{-5}$ g.[3] We wish now to point out that the prescription (3.4) is not as ad hoc as it may appear. In the Einstein–Cartan theory of gravitation, which must replace Einstein's theory for matter at high density (Heyl et al., 1974), the metrical energy-momentum stress tensor $\sigma^{\alpha\beta}$ of spinning matter is modified by a universal gravitational contact spin–spin interaction[4]

$$G^{\alpha\beta} = -\kappa\sigma'^{\alpha\beta} \tag{3.6}$$

where

$$\begin{aligned}\sigma'^{\alpha\beta} = \sigma^{\alpha\beta} &+ [\tau^{\alpha\beta\sigma} + \tau^{\beta\alpha\sigma} + \tfrac{1}{2}(g^{\sigma\alpha}\tau^{\beta} + g^{\sigma\beta}\tau^{\alpha} - g^{\alpha\beta}\tau^{\sigma})]\partial_\sigma[\log(-g)^{1/2}] \\ &+ \kappa\{\tau^{\alpha}\tau^{\beta} + \tau_{\lambda\cdot}^{\alpha\sigma}\tau_{\sigma\cdot}^{\beta\lambda} + 4\tau_{\cdot\cdot}^{\sigma\alpha\nu}\tau_{[\nu\cdot\sigma]}^{\cdot\beta} + \tfrac{1}{2}g^{\alpha\beta}[3\tau_\lambda\tau^\lambda + 2\tau_{\sigma\cdot}^{\lambda\nu}(\tau_{\nu\cdot\lambda}^{\cdot\sigma} + 2\tau_{\nu\lambda}^{\sigma})]\}\end{aligned} \tag{3.6a}$$

($\tau_{\lambda\cdot}^{\alpha\beta}$ is the canonical spin angular momentum tensor density and $\tau^{\alpha\beta\sigma} = g^{\sigma\lambda}\tau_{\lambda\cdot}^{\alpha\beta}$, $\tau_{\lambda\nu}^{\alpha} = g_{\nu\beta}\tau_{\cdot\lambda}^{\alpha\beta}$ and $\tau_\lambda = \tau_{\lambda\nu}^{\alpha}$).

Thus

$$G_{\alpha\beta} + fg_{\alpha\beta} = -\kappa\tilde{\sigma}_{\alpha\beta} \tag{3.7}$$

where

$$f = \tfrac{1}{2}\kappa\{-\tau^\sigma\partial_\sigma[\log(-g)^{1/2}] + \kappa[3\tau_\lambda\tau^\lambda + 2\tau_{\sigma\cdot}^{\lambda\nu}(\tau_{\nu\cdot\lambda}^{\cdot\sigma} + 2\tau_{\nu\lambda}^{\sigma})]\} \tag{3.8}$$

[3] Observe that unitons, with the Planck mass, therefore cannot be regarded as wavelike perturbations of the background Euclidean geometry; they are not wave-mechanical fields, from a geometrodynamical point of view, that is.

[4] Our expression, though equivalent, differs in form slightly from that given by Heyl et al. because of a difference in our definition of the affine connection whereby covariant indices are transposed.

and

$$\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} + [\tau_{\alpha\cdot\beta}^{\cdot\sigma} + \tau_{\beta\cdot\alpha}^{\cdot\sigma} + \tfrac{1}{2}(\delta_{\beta}{}^{\sigma}\tau_{\alpha} + \delta_{\alpha}{}^{\sigma}\tau_{\beta} - g_{\alpha\beta}\sigma^{\sigma})]\partial_{\sigma}[\log(-g)^{1/2}] + \kappa(\tau_{\alpha}\tau_{\beta} + \tau_{\lambda\cdot\alpha}^{\cdot\nu}\tau_{\nu\cdot\beta}^{\cdot\lambda} + 4\tau_{\alpha\cdot}^{\lambda\nu}\tau_{[\lambda\beta\nu]}^{\cdot\cdot}) \tag{3.8a}$$

Now for matter at the Planck density, the volume $L_0{}^3$ of space has spin angular momentum $\hbar$. Hence

$$\tau_{\lambda\cdot}^{\alpha\beta} \sim c\hbar/L_0{}^3 \qquad \text{and} \qquad f \sim \kappa^2\tau_{\sigma\cdot}^{\lambda\nu}\tau_{\nu\lambda}^{\sigma} \sim L_0^{-2} \tag{3.9}$$

The gravitational self-interaction due to coupling of spin to space-time torsion induces a cosmological space-time-dependent term with

$$f(x) = \Lambda(x) = a(x)/L_0{}^2 \qquad [a(x) \text{ dimensionless}, \sim 1] \tag{3.10}$$

Notice that the second term in (3.8) is bilinear in the spin density and does not, unlike the first, vanish when the energy-momentum tensor is averaged over the ensemble of randomly orientated particle spins ($\sim 10^{40}/\text{cm}^3$ for protons packed together by "strong" gravity at the Planck density). So the cosmological term cannot be ignored when considering matter at the highest density possible. Of course, for the classical vacuum and prequantized world, it vanishes. But in the real vacuum, space is not empty and it persists. In support of Nickerson's hypothesis, it is suggested that the vacuum is a plenum with the Planck density and elementary particles are a perturbation of its gravitational field. Then in the absence of matter, $g_{\mu\nu} = \eta_{\mu\nu}$ and $G^{\alpha\beta} = 0$ so that

$$\langle 0|f|0\rangle\eta_{\alpha\beta} = -\kappa\langle 0|\sigma_{\alpha\beta}^{vac} + \kappa(\tau_{\alpha}\tau_{\beta} + \tau_{\lambda\cdot\alpha}^{\cdot\nu}\tau_{\nu\cdot\beta}^{\cdot\lambda} + 4\tau_{\alpha\cdot}^{\lambda\nu}\tau_{[\lambda\beta\nu]}^{\cdot\cdot})|0\rangle$$

or

$$(a/L_0{}^2)\eta_{\alpha\beta} = -\kappa\Sigma_{\alpha\beta}^{vac}$$

with

$$a = \langle 0|a(x)|0\rangle = \tfrac{1}{2}\langle 0|3\hat{\tau}_{\lambda}\hat{\tau}^{\lambda} + 2\hat{\tau}_{\sigma\cdot}^{\lambda\nu}(\hat{\tau}_{\nu\cdot\lambda}^{\cdot\sigma} + 2\hat{\tau}_{\nu\lambda}^{\sigma})|0\rangle \qquad (\hat{\tau}_{\nu\lambda}^{\sigma} \equiv \kappa L_0\tau_{\nu\lambda}^{\sigma})$$

$$\Sigma_{\alpha\beta}^{vac} = \langle 0|\sigma_{\alpha\beta}^{vac} + \kappa S_{\alpha\beta}^{vac}|0\rangle$$

and

$$S_{\alpha\beta}^{vac} = \tau_{\alpha}\tau_{\beta} + \tau_{\lambda\cdot\alpha}^{\cdot\nu}\tau_{\nu\cdot\beta}^{\cdot\lambda} + 4\tau_{\alpha\cdot}^{\lambda\nu}\tau_{[\lambda\beta\nu]}^{\cdot\cdot}$$

(It must be emphasized that the perspective is geometrodynamic, with $g_{\mu\nu}$ the gravitational field of the dense vacuum which is weakly perturbed by matter. This is a reasonable view since the Planck density is 10^{76} times as large as elementary particle densities.) We have thus derived Nickerson's hypothesis from the Einstein–Cartan theory of gravitation. The negative finite expectation value of the energy of a spin-1 Bose sea at the Planck density may indeed generate a gravitational field of universally Euclidean metric. Observe from (3.8) that $\Lambda \equiv \langle 0|f|0\rangle$ is positive, as is necessary for bosons of negative energy. Indeed, the requirement of a negative energy density for the vacuum fixes the sign of the gravitational coupling constant and so implies that gravity is an attractive force. Because the cosmological term is missing in Einstein's preferred equations, his theory cannot predict this common fact of experience, as is well known. However, the difficulty present in Nickerson's suggestion is that in the attempt to derive wave mechanics as the geometrodynamics of vacuum energy fluctuations and *also* to retain Einstein's preferred equations, one is faced with two inequivalent choices for the vacuum energy-momentum stress tensor that serves as the source of the dominant Euclidean geometry of empty space. One choice,

$$\Sigma_{\mu\nu}^{vac} = -(\Lambda/\kappa)g_{\mu\nu} \tag{3.11}$$

leads to Einstein's preferred equation

$$G^{\mu}_{\cdot\nu} = -\kappa T^{\mu}_{\cdot\nu}$$

and energy-momentum conservation of matter, because of Bianchi's identity. But no linear second-order wave-mechanical equation results because of the cancellation of the essential term leading to oscillations with the wavelength L_0. The other possibility

$$\Sigma_{\mu\nu}^{vac} = -(\Lambda/\kappa)\eta_{\mu\nu} \tag{3.12}$$

leads to wave mechanics but also to Einstein's equation with a cosmological term:

$$G_{\mu\nu} + \Lambda\gamma_{\mu\nu} = -\kappa T_{\mu\nu} \tag{3.13}$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} \tag{3.14}$$

The metric condition[5] $\nabla_\lambda g_{\mu\nu} = 0$ implies $\nabla_\lambda \gamma_{\mu\nu} \neq 0$ so that energy-momentum of matter alone is not conserved but rather that of matter *and* the vacuum:

$$\nabla_\nu \Sigma^{\nu}_{\cdot\mu} = 0 \tag{3.15}$$

$$\nabla_\nu \Sigma^{\nu\,vac}_{\cdot\mu} = -\nabla_\nu T^{\nu}_{\cdot\mu} \tag{3.16}$$

We must recognize that even as the *classical limit of quantum geometrodynamics Einstein's preferred equation is invalid*, for the vacuum 3-force

$$f_\kappa \equiv \nabla_\nu T^{\nu}_{\cdot\kappa} = -\nabla_\nu \Sigma^{\nu\,vac}_{\cdot\kappa} \tag{3.17}$$

due to nonconservation of matter is nonvanishing when $\hbar \to 0$. Since the geodesic equation of motion of a test particle follows from the assumption of conservation of energy-momentum, the world trajectory of a particle whose energy-momentum is not conserved is not the solution $x^\mu = \xi^\mu(s)$ of

$$\frac{d^2x^\mu}{ds^2} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} \frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0$$

[5] ∇_λ denotes covariant differentiation with respect to the Christoffel symbol.

Instead, if we consider, for simplicity, a point particle of mass m whose energy-momentum tensor is given by

$$(-g)^{1/2}T^{\mu\nu} = \rho \frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds} \tag{3.18}$$

where

$$\rho = m\delta^4(x - \xi) \tag{3.19}$$

and integrating (3.12) over a world tube enclosing its trajectory, we obtain

$$\frac{d^2\xi^{\mu}}{ds^2} + \left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} \frac{d\xi^{\alpha}}{ds}\frac{d\xi^{\beta}}{ds} = -\frac{1}{m} F^{\mu}_{vac}(\xi) \tag{3.20}$$

where

$$F^{\mu}_{vac}(\xi) = \int (-g)^{1/2}\, d^3x \nabla_{\nu} \Sigma^{\mu\nu}_{vac} \tag{3.21}$$

and

$$\nabla_{\nu}\Sigma^{\mu\nu}_{vac} = \frac{\partial \rho^{\mu}}{\partial \Omega} = \frac{1}{(-g)^{1/2}} \frac{df^{\mu}_{vac}}{d\tau} \qquad \left(f^{\mu}_{vac} = \frac{d\rho^{\mu}}{dt} \right) \tag{3.22}$$

so that

$$F^{\kappa}_{vac}(\xi) = \frac{3}{2}\frac{c^4}{G} u^{\kappa} n \qquad (u_{\kappa}u^{\kappa} = 1) \tag{3.23}$$

The infinite set of values of n implies (for a Dirac particle at least) an infinite family of possible world trajectories that are no longer geodesics; energy nonconservation ($n \neq 0$) implies indeterminism, for only $n = 0$ corresponds to classical gravitation in which energy-momentum is conserved and the vacuum has no dynamical properties. Equation (3.12) is preferred over (3.11). Suppose now that the 3-acceleration $d^2\xi^{\kappa}/ds^2 \ll c/T_0$. Then from (3.20) and (3.23), $m \gg M_0$, which verifies [see (3.5) et seq.] Nickerson's conclusion that the Planck mass represents, *effectively*, a lower limit on the masses of particles whose motion can be described by classical deterministic mechanics. Particles heavier than M_0 are less accelerated than lighter particles by Planck fluctuations of the vacuum so their motion is less random and more "classical." Also, according to the Dirac veto

$$\Psi(D) = 0, \qquad g_{\mu\nu}(D) = 0 \tag{3.24}$$

along the world-line of dual matter. Thus

$$T_{\mu\nu}(D) = 0 \tag{3.25}$$

This implies, from (3.13), $\gamma_{\mu\nu} = 0$ and from (3.14), $\eta_{\mu\nu} = 0$ along D. The latter result means our choice is consistent with the results [equations (7.7) and (7.8)] found by Riegert. The chosen equation,

$$G_{\mu\nu} + \Lambda\gamma_{\mu\nu} = -\kappa T_{\mu\nu}$$

reduces to the trivial identity $0 = 0$ along D. Einstein's preferred equation merely to the vacuum field equations

$$R_{\mu\nu}(D) = 0$$

which does not necessarily imply the correct solution $g_{\mu\nu} = 0$. The presence of a cosmological term together with the result of Riegert that the curvature tensor vanishes along D *forces* the metric tensor to vanish as well. From (3.12) and (3.24) we conclude

$$\Sigma_{\mu\nu}(D) = \Sigma_{\mu\nu}^{vac}(D) = 0 \tag{3.26}$$

(3.24) and (3.26) can be understood as follows: Particles, geometrodynamically, are permanent (insofar as they are stable), highly localized weak deformations of the vacuum Euclidean geometry. We may say that rest mass is the potential energy stored in such deformations. Because this stress in the otherwise flat space-time continuum is communicated to the universe at large, the cosmic boundary conditions (mass, radius, age, etc.) determine their amplitude (mass) in the spirit of Mach's principle. Since, by definition, particles "see" the vacuum as Euclidean, they do not gravitationally interact with it. Instead, they constitute inertial frames with respect to the vacuum state. On the other hand, the world-lines D are null geodesics referred to a coordinate frame that is inertial with respect to the vacuum state and so the gravitational field of the *vacuum*, namely, the Minkowski metric, must vanish in coordinate frames free-falling with respect to it, i.e., the quantized singularities of the gravitationally perturbed vacuum move on the geodesics of the matter comprising the vacuum. The Dirac field must disappear at all points along them, for matter cannot gravitationally interact with the vacuum, the 3-force arising from energy nonconservation being noninertial in origin. Only matter gravitationally interacting with the vacuum could exist at these points. We can only conclude that the energy-nonconserving sources of the dual Riemann tensor are just "vacuum" matter itself and that the force is due to the creation of gravitational charges, i.e., unitons, in support of the conclusions of Section 2.

From (3.3) and (3.12) it can be seen that when matter is itself near the Planck density, then

$$\Sigma_{\mu\nu} \simeq 0$$

so

$$G_{\mu\nu} + \Lambda g_{\mu\nu} \simeq 0$$

that is, a truly empty quasi-de Sitter space of (approximately) constant curvature $R = 4a/L_0^2$ and radius of curvature $\sim L_0$ is the final geometry of matter in the last stage of gravitational collapse when equilibrium has been reached with the balancing of attractive forces, due to the coupling of mass to curvature, and repulsive spin–spin contact forces due to the coupling between spin and torsion of space-time. We may speculate that the initial state of the universe (and its final destiny, if it is closed) is a void, in which the distinction between the vacuum and matter (Planck fluctuations are of the order of unity) becomes meaningless.[6]

Further support to the interpretation above that the vacuum is a dense uniton sea comes from the following considerations: According to the Einstein–Cartan theory of gravitation, the divergence of the canonical energy-momentum stress tensor (see Appendix A) is

$$\nabla_\mu {}^{+}\Sigma^{\mu}_{\cdot\alpha} = \tau^{\sigma\lambda}_{\beta\cdot} R^{\beta}_{\lambda\sigma\alpha} \tag{3.27a}$$

or

$$\nabla_\mu \Sigma^{\mu}_{\cdot\alpha} = \tau^{\sigma\lambda}_{\beta\cdot} R^{\beta}_{\lambda\sigma\alpha} + 2\kappa(\tau_\mu \Sigma^{\mu}_{\cdot\alpha} - \tau^{\lambda}_{\mu\alpha}\Sigma^{\mu}_{\cdot\lambda} - \tfrac{1}{2}\tau_\alpha \Sigma) \tag{3.27b}$$

where $\Sigma = \Sigma^{\alpha}_{\cdot\alpha}$. Suppose the matter is made up of spin-1 bosons, mass m, occupying an average volume $\sim L^3$; then

$$\tau\kappa \sim c\hbar/L^3 \qquad \text{and} \qquad \Sigma^{\sigma}_{\cdot\sigma} \sim mc^2/L^3 \sim \Sigma$$

Arbitrarily choosing the last term in (3.27) as giving typical order of magnitude, the 3-force density is

$$f_\kappa \equiv \nabla_\mu \Sigma^{\mu}_{\cdot\kappa} \sim \kappa\tau_\kappa \Sigma \sim \frac{G\hbar}{c}\cdot\frac{m}{L^\mu} \tag{3.28}$$

Assuming the force density is uniform over L, the interboson force is

$$F_\kappa \sim L^3 f_\kappa \sim \frac{G\hbar}{c}\cdot\frac{m}{L^3}$$

Identifying the particles with unitons, then $m \sim (\hbar c/G)^{1/2}$ and $F_\kappa \sim c^4/G$ if $L \sim L_0$. The repulsive force between unitons due to their spins is of the same order of magnitude as their mutual Newtonian attractive force when they are at the Planck distance apart and gravitational equilibrium at the Planck density is possible in principle. In Appendix B, it is shown that the rate of change of the (contracted) energy-momentum tensor density is

$$\partial_\mu \Sigma = 2[\tau^{\sigma\lambda}_{\alpha\cdot} R^{\alpha}_{\lambda\sigma\mu} + \kappa(\tau_\lambda \Sigma^{\lambda}_{\cdot\mu} - \tfrac{1}{2}\tau_\mu \Sigma)] \tag{3.29}$$

so

$$\frac{\partial}{\partial t}\Sigma \sim \kappa c \tau_\lambda \Sigma^{\lambda}_{\cdot\sigma} \sim c f_\kappa$$

The energy content of a Planck volume changes at a rate

$$\frac{dE}{dt} \sim \frac{\partial}{\partial t}(L_0{}^3\Sigma) \sim cF_\kappa \sim \frac{c^5}{G} \tag{3.30}$$

[6] One is reminded of the Sunya (Void) doctrine of Mahayana Buddhism.

which is the same as the maximum rate at which the vacuum force F_κ does work. In addition, from the Einstein–Cartan field equations for spinning matter:

$$G_{\mu\nu} = -\kappa\Sigma_{\mu\nu} \tag{3.31}$$

where $G_{\mu\nu}$ is the generalized asymmetric Einstein tensor, we have

$$R = \kappa\Sigma \tag{3.32}$$

Assuming the uniton mass M is effectively distributed over its Compton wavelength $\lambda_c = L_0/n^{1/2}$, then $\Sigma \sim mc^2/\lambda_c{}^3$ where $Gm^2 = n\hbar c$ and so

$$R \sim n^2/L_0{}^2 \tag{3.33}$$

Hence U–$\bar{U}$ creation causes quantized ripples in the local Euclidean geometry with characteristic radius of curvature L_0/n. U–$\bar{U}$ creation is consistent with Heisenberg's uncertainty principle for

$$\Delta(R^{-1/2}) \sim L_0, \qquad \Delta t \sim \frac{1}{c}\,\Delta(R^{-1/2}) \sim \frac{L_0}{c}, \qquad \Delta E \sim M_0c^2 = \frac{\hbar c}{L_0}$$

so

$$\Delta E \Delta t \sim \hbar$$

and

$$\Delta P \sim F\Delta t \sim \frac{c^4}{G}\cdot\frac{L_0}{c} = \frac{c^3L_0}{G}, \qquad \Delta x \sim L_0$$

so

$$\Delta P \Delta x \sim \frac{c^3L_0{}^2}{G} \sim \hbar$$

A stronger statement would be that the *uncertainty principle itself is a consequence of stochastic fluctuations on the Planck scale of the curvature of empty space.* According to (3.30), the minimum exchange of energy between matter and the vacuum in the shortest time $\Delta t \sim T_0$ is

$$\Delta E = \frac{dE}{dt}\,\Delta t \sim n\frac{c^5}{G}T_0$$

therefore

$$\Delta E \Delta t \sim n\frac{c^5}{G}T_0{}^2 \sim n\hbar \geqslant \hbar \tag{3.34}$$

As we stated earlier, $n = 0$ corresponds to conserved energy-momentum for matter and no uncertainty in either E or t, i.e., classical physics. $n \neq 0$ implies nonconservation, a nonclassical dynamic vacuum and the uncertainty principle. A statistical description is rendered necessary when particles have masses much less than 10^{-5} g or 10^{19} GeV because their motion is randomized to a significant degree by Planck fluctuations in the geometry. Einstein's equation

$$G_{\mu\nu} = -\kappa T_{\mu\nu}$$

is valid when the mass of gravitating matter is large compared with the Planck mass, its fluctuating coupling to the vacuum being negligible compared with its energy. The background "noise" of the random energy density fluctuations, of order of magnitude M_0c^2 would represent a minute perturbation of the total energy of macroscopic particles so that classical mechanics would be an excellent approximation. However, for lighter particles, the vacuum coupling would be increasingly important, until on the atomic and nuclear scale, stochasticity of the fine-grained geometry of space generates ensembles of particle trajectories and hence particle *fields*. We could say that the Planck mass divides bodies into effectively classical and quantum objects. For microscopic particles, Einstein's preferred equation has to be replaced by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa \Sigma_{\mu\nu}$$

with the Λ term guaranteeing that matter free space is Euclidean and implying the wave-mechanical nature of such particles. This equation would be the expectation value of the appropriate field operator equation. Of course, until Planck constant has itself been derived in terms of the constants of classical physics, it cannot be said that a classical or neoclassical derivation of quantum mechanics is possible. The inherent reason for the existence of an intrinsic Planck scale cutoff in energy and distance is connected with the unobservability of events inside the event horizon $r \sim L_0$ of black hole unitons whose mass represent the lower limit on the applicability or validity of unquantized Einsteinian geometry (classical space-time) to particle dynamics in a space-time continuum whose geometry is itself subject to the uncertainty principle. Physical insight into the role of unitons in vacuum physics is possible from the analogy of U–$\bar{U}$ states with excitons in crystals. When photons of energy less than the energy gap are absorbed by a crystal, electrons can be excited from the filled valence band to form stable electron–hole pairs. The analogy is complete if we regard the vacuum as the valence band and matter as the conduction band of electrons. The analogy is all the more striking when we consider that many authors have suggested a stochastic (Markovian) origin of quantum mechanics. The uncertainty principle can be derived assuming the vacuum has an effective diffusion constant $D = \hbar/2m$ for particles of mass m. For unitons this gives the relation

$$L_0 = (D \cdot 2T_0)^{1/2} \tag{3.35}$$

which is similar (Kittel, 1967) to

$$d_p = (D\tau_p)^{1/2}, \qquad d_n = (D\tau_n)^{1/2}$$

which connects the diffusion length d_p (d_n) and hole (electron) lifetime τ_p (τ_n) in n-type (p-type) semiconductors. L_0 and $2T_0$ can be thought of as the diffusion length and lifetime, respectively, of unitons before radiative recombination into gravitons. An investigation for the future should be to see whether Planck's constant could be derived from the statistical thermodynamics of an ensemble of colliding black holes with the Planck mass as an adjustable parameter setting thermodynamic equilibrium.

It must be pointed out that energy nonconserving processes should not be thought of as the occasional creation of a stationary particle at isolated points in space. Rather space scintillates with randomly occurring points at which sudden fluctuations of the vacuum energy density take place. These are temporary sources and sinks of quantized "dual" gravitational charges whose fields exert quantized forces on matter during the time T_0 they exist. Only the incoherence of such events prevents the gradual accumulation of energy in space. So there is no creation continuously increasing the mass of the universe. Since in (3.34) both positive and negative values of n are allowed, the energy of matter can either increase or decrease. In stochastic theories of quantum mechanics, the relaxation time τ during which initial, sharp motion of a classical particle of mass m disperses into a statistical ensemble of possible trajectories and the average energy approaches the measured expectation value is

$$\tau = \frac{\hbar}{mc^2} = \frac{M_0}{m} T_0$$

For electrons, $M_0/m \sim 10^{22}$, and we have to wait $\sim 10^{-21}$ sec or the duration of 10^{22} Planck fluctuations before equilibrium is reached, that is 10^{23} times as long as we would have to wait in order to measure the energy of a particle with the Planck mass, which can be measured only after 10^{-44} sec after the instant it had that energy. Of course, the uncertainty principle is invalid before this relaxation time is up, so quantum electrodynamics can be predicted to break down for processes involving electrons that last less than 10^{-21} sec (10^{-24} sec for protons). The hadron resonances, accordingly, represent, the limit of applicability of quantum mechanics.

4. STRONG INTERACTIONS AS THE GRAVITATIONAL COUPLING OF "DUAL" MATTER

Strong forces with a range typically of 10^{-13} cm produce shifts in energy ~ 100 MeV or 10^{-4} ergs. This production of energy corresponds to an average force of $\sim 10^{11}$ dynes. The quantized force acting on matter due to energy nonconservation (also the force between gravitational charges separated by the Planck distance) is, as we have seen, $\sim c^4/G \sim 10^{49}$ dynes. Now in "strong" gravity theories, the weak (Newtonian) gravitational coupling constant of long-range gravity, G_N, is replaced by the strong gravity constant of short-range gravity, G_s, which is connected to the former by $G_s \sim 10^{38} G_N$. The order of magnitude of the strong-coupling strength is correctly predicted if the interaction is identified with the coupling of "strong" gravitational charges whose quantized masses are now given by

$$G_s M^2 = n\hbar c \tag{4.1a}$$

that is,

$$M = n^{1/2} M_0 \qquad (M_0 = 1.22 \text{ GeV}) \tag{4.1b}$$

Justification beyond that usually given for this replacement of constants can be given by hypothesizing that the "strong" gravitational field equations for hadrons are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa_N \Sigma_{\mu\nu}^{vac} - \kappa_s T_{\mu\nu} \tag{4.2}$$

where $\kappa_s = 8\pi G_s/c^4$. Then (3.17) and (3.23) become

$$f_\kappa = \nabla_\nu T^{\nu}_{\cdot\kappa} = -\frac{G_N}{G_s} \nabla_\nu \Sigma^{\nu\, vac}_{\cdot\kappa}$$

and

$$F^{\kappa}_{vac} = \frac{3}{2} \frac{G_N}{G_s} \frac{c^4}{G_N} n = \frac{3}{2} \frac{c^4}{G_s} n \tag{4.3}$$

Also, the rate of nonconservation of energy is $\sim c^5/G_s \sim 10^{23}$ GeV sec^{-1}. During the time scale of strong interactions $\sim 10^{-24}$ sec, the energy change would be ~ 100 MeV, which is exactly the hadronic scale set by strong forces. Actually, the time scale is also predicted, since the "strong" Planck length $L_0 = 0.16$ F is the uniton Compton wavelength which would determine the range of "strong" gravity and so the energy of interaction as well. Alternatively, it can be seen as the "strong" Planck time $T_0 = L_0/c \sim 10^{-24}$ sec. This interpretation correctly predicts the essential features—strength, duration, and range—that distinguish strong forces from the weak and electromagnetic. Since leptons do not interact strongly, they do not couple to "strong" unitons (mass M_s) but only to "weak" unitons (mass $M_w = 10^{19} M_s$). The coupling between "strong" and "weak" unitons is of the same order of magnitude when they are the same distance apart, so vacuum polarization into massive "weak" unitons might be expected to be important in strong interactions. However, such Feynmann diagrams involving $\sim 10^{20}$ Planck fluctuations, each of $\sim 10^{19}$ GeV, during the interaction, would have an infinitesimal contribution to the overall transition amplitude. It should be emphasized that, in assuming vacuum polarization to occur on two levels—one the "weak" Planck level responsible for quantization and the other the "strong" Planck scale that is the arena for strong forces—we are not implying a bimetric theory of gravitation. All hadrons are sources of one gravitational field which is made up of two components. Leptons, without "strong" gravitational charge, generate and couple to the long-range Newtonian field, whilst hadrons, as gravitational dyons, couple to both the Newtonian and Yukawa potentials, the latter arising from exchange of massive U–$\overline{U}$ states, i.e., for a gravitational dyon

$$g_{00} = 1 + \frac{2\phi}{c^2} \tag{4.4}$$

and

$$\phi = \phi_N + \phi_s \tag{4.5}$$

where

$$\phi_N = -\frac{G_N M}{r}, \qquad \phi_s = -\frac{G_s M}{r} e^{-r/R} \tag{4.6}$$

and from (2.11)

$$R = \lambda_c \sim L_0$$

Weak gravity would dominate ($\phi_N > \phi_s$) for $e^{r/L_0} > 10^{38}$, that is for $r > 90L_0 \sim 14$ F. Over a distance of about 10 Compton wavelengths, the character of strongly interacting particles would change rapidly from leptonic to hadronic. It should be noticed that uniton–antiuniton pairs, coupled by a Yukawa gravitational potential, cannot form Bohr-type stationary bound states. This is seen readily by considering two unitons of the same mass M, revolving in circular orbits of radius r about their common center of mass with angular velocity ω. The force per unit mass is

$$F = -\frac{d\phi}{dr} = F_N + F_s \tag{4.7}$$

where

$$F_N = -\frac{G_N M}{r^2}, \qquad F_s = -\frac{G_s M}{r^2}\left(1 + \frac{r}{R}\right)e^{-r/R}$$

so

$$F_s \gg F_N \qquad \text{provided} \qquad r \sim R$$

The centripetal acceleration of each uniton $\omega^2 r$ is due to the gravitational force

$$F = M\omega^2 r = M|F| \simeq M|F_s| = \frac{G_s M^2}{(2r)^2}\left(1 + \frac{2r}{R}\right)e^{-2r/R} \tag{4.8}$$

The quantized orbital angular momentum is

$$Mr^2\omega + Mr^2\omega = 2Mr^2\omega = n\hbar \qquad (n = 1, 2, 3, \ldots) \tag{4.9}$$

Using (4.1b), the radii r_n of the orbitals are given by the roots of the transcendental equation

$$X(1 + X) = 2ne^x \qquad [X = 2(n)^{1/2}r_n/L_0]$$

none of which are positive. U–$\overline{U}$ states are unlikely to be long-lived and would exist only as spin-2 resonances. On the other hand, the remarks made in Section 3 [see (3.28)] suggest that such "unitonium" states might be stabilized (though no longer Bohr orbitals) by the repulsive spin–spin interaction which could prevent collapse and annihilation into gravitons. Suppose $F \sim c^4/G_s$, then instead of (4.8) we have

$$\frac{G_s M^2}{(2r)^2}\left(1 + \frac{2r}{R}\right)e^{-2r/R} \sim \frac{c^4}{G_s}$$

and stable states may exist since there are real positive roots of the resulting transcendental equation

$$X^2 e^x = n^2(1 + X)$$

[for $n = 1$, $r_1 = 0.4L_0$, comparing with the value $r_s = 0.8L_0$ of the strong gravity Schwarzschild radius r_S obtained as solution of $g_{00}(r) = 0$]. If the boson unitons are spinless, however, the spin–spin interaction vanishes and U–$\overline{U}$ states bound by strong gravity must be unstable. Semiclassical calculations such as those above are valid insofar as the unitons have the least mass for classical mechanics to be valid, as discussed earlier. The Planck mass also represents the least mass for a particle obeying Einstein's field equations, i.e., they are valid only if

$$|T_{\mu\nu}| \gg |\Sigma_{\mu\nu}^{vac}|$$

or

$$\frac{mc^2}{\lambda_0^{\,3}} \gg \kappa^{-1}L_0^{-2}$$

i.e., $m \gg M_0 \sim 10^{-5}$ g for leptons (10^{-24} g for hadrons).

5. MASSIVE GRAVITONS AND HADRON GEOMETRY

We now show that the gravitational field equations proposed above (the vacuum modified Einstein equations) reduce, in the case of hadrons, to the wave equation of a massive pure spin-2 field (in the weak-field approximation) whose static, spherically symmetric solution has the Yukawa form. Let

$$g_{\mu\nu} = \eta_{\mu\nu} + \xi_{\mu\nu} \qquad (|\xi_{\mu\nu}| \ll 1) \tag{5.1}$$

so that

$$g^{\mu\nu} = \eta^{\mu\nu} - \xi^{\mu\nu}$$

We propose that hadrons obey the following gravitational field equation:

$$G_{\mu\nu} + \Lambda_s g_{\mu\nu} = -\kappa_s \Sigma_{\mu\nu}^{vac} - \kappa_s T_{\mu\nu} \tag{5.1a}$$

or

$$G_{\mu\nu} + \Lambda_s \xi_{\mu\nu} = -\kappa_s T_{\mu\nu} \tag{5.1b}$$

where $\Lambda_s \sim L_0^{-2}$ and $L_0 = (G_s\hbar/c^3)^{1/2}$ is the Planck length of "strong" gravity ($L_0 = 0.16$ F). The scalar curvature R is, to first order in $\xi_{\mu\nu}$,

$$R = \Lambda_s \xi + \kappa_s T \qquad (\xi = \xi_\nu{}^\nu, \quad T = T_\nu{}^\nu) \tag{5.2}$$

The Ricci tensor is

$$R_{\mu\nu} = -\kappa_s(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T) - \Lambda_s(\xi_{\mu\nu} - \tfrac{1}{2}\xi\eta_{\mu\nu}) \tag{5.3}$$

Now

$$2R_{\mu\nu} = \Box^2 \xi_{\mu\nu} - \partial_\mu \tau_\nu - \partial_\nu \tau_\mu \tag{5.4}$$

where

$$\tau_\mu = \partial_\lambda \xi_\mu{}^\lambda - \tfrac{1}{2}\partial_\mu \xi$$

In the presence of matter, the wave equation for the perturbed metric is

$$(\Box^2 + 2\Lambda_s)\xi_{\mu\nu} = -2\kappa_s T_{\mu\nu} + (\Box^2 \xi - \partial_\lambda \partial_\sigma \xi^{\lambda\sigma})\eta_{\mu\nu} + \partial_\mu \tau_\nu + \partial_\nu \tau_\mu \tag{5.5}$$

and in vacuo

$$(\Box^2 + 2\Lambda_s)\xi_{\mu\nu} = \partial_\mu \tau_\nu + \partial_\nu \tau_\mu + \Lambda_s \xi \eta_{\mu\nu} \tag{5.6}$$

The last term is the deviation from the Einstein linearized vacuum field equations. Consider an infinitesimal change of coordinates

$$x^\alpha \to x^{\alpha\prime} = x^\alpha + \epsilon^\alpha(x) \tag{5.7}$$

From the tensor property of $g_{\mu\nu}$

$$g'_{\mu\nu}(x') = \frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} g_{\lambda\sigma}(x)$$

we have

$$\xi'_{\mu\nu} = \xi_{\mu\nu} - \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu \tag{5.8a}$$

and

$$\xi' = \xi - 2\partial_\lambda \epsilon^\lambda \tag{5.8b}$$

so that

$$\Box'^2 \xi'_{\mu\nu} = \Box^2 \xi'_{\mu\nu} = \Box^2(\xi_{\mu\nu} - \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu) \tag{5.9}$$

(5.5) can be written

$$(\Box^2 + 2\Lambda_s)\tilde{\xi}_{\mu\nu} = -2\kappa_s(T_{\mu\nu} - \tfrac{1}{2}Tg_{\mu\nu}) - \tfrac{1}{2}\eta_{\mu\nu}(\kappa_s T + \partial_\lambda \tau^\lambda) + \partial_\mu \tau_\nu + \partial_\nu \tau_\mu \tag{5.10}$$

where

$$\tilde{\xi}_{\mu\nu} \equiv \xi_{\mu\nu} - \tfrac{1}{4}\xi\eta_{\mu\nu} \qquad (\tilde{\xi} \equiv \tilde{\xi}_\nu{}^\nu = 0) \tag{5.11}$$

The vacuum field equations are

$$(\Box^2 + 2\Lambda_s)\tilde{\xi}_{\mu\nu} = \partial_\mu \tau_\nu + \partial_\nu \tau_\mu - \tfrac{1}{2}\eta_{\mu\nu}\partial_\lambda \tau^\lambda \tag{5.12a}$$

and

$$(\Box^2 - 2\Lambda_s)\xi = 2\partial_\lambda \tau^\lambda \tag{5.12b}$$

Using (5.11), (5.12b) becomes

$$(\Box^2 - \tfrac{4}{3}\Lambda_s)\xi = \tfrac{4}{3}\partial_\mu \partial_\lambda \tilde{\xi}^{\mu\lambda} \tag{5.13}$$

$\tilde{\xi}_{\mu\nu}$ has five independent components if we impose the divergence condition

$$\partial_\lambda \tilde{\xi}^{\mu\lambda} = 0 \tag{5.14a}$$

that is,

$$\partial_\lambda \xi^{\mu\lambda} = \tfrac{1}{4}\partial^\mu \xi \tag{5.14b}$$

or

$$\tau_\mu = -\tfrac{1}{4}\partial_\mu \xi \tag{5.14c}$$

The Lorentz gauge condition $\tau_\mu = 0$, which implies massless gravitons, is no longer possible because $\Lambda \neq 0$. The new gauge condition (5.14b) is covariant under infinitesimal coordinate changes provided

$$\Box^2 \epsilon^\mu + \tfrac{3}{2}\partial^\mu \partial_\lambda \epsilon^\lambda = 0 \tag{5.15}$$

(5.8b) and (5.14c) lead to the transformation law

$$\tau'_\nu = \tau_\nu + \tfrac{1}{2}\partial_\nu \partial_\lambda \epsilon^\lambda = \tau_\nu - \tfrac{1}{3}\Box^2 \epsilon_\nu \tag{5.16}$$

(5.6) and (5.9) give

$$\tau'_\nu = \tau_\nu - (\Box^2 - 2\Lambda_s)\epsilon_\nu \tag{5.17}$$

so that

$$(\Box^2 - 3\Lambda_s)\epsilon_\nu = 0 \tag{5.18}$$

From (5.12b) and (5.14c),

$$(\Box^2 - \tfrac{4}{3}\Lambda_s)\xi = 0 \tag{5.19}$$

It is consistent with this result to put the right-hand side of (5.12a) equal to zero, since then

$$\partial_\mu \partial_\nu \xi = \tfrac{1}{3}\Lambda_s \xi \eta_{\mu\nu}$$

and so

$$(\Box^2 - \tfrac{4}{3}\Lambda_s)\xi = 0$$

which is (5.19). In summary, the weak-field approximation of (5.1a) leads, in vacuo, to the field equations of a massive spin-2 field with five independent components:

$$(\Box^2 + 2\Lambda_s)\tilde{\xi}_{\mu\nu} = 0 \tag{5.20}$$

where

$$\tilde{\xi}_{\mu\nu} = \xi_{\mu\nu} - \tfrac{1}{4}\xi\eta_{\mu\nu} \qquad \tilde{\xi} = 0 \qquad \partial_\nu\tilde{\xi}^{\mu\nu} = 0$$

The spherically symmetric, static solution for a stationary point mass situated at the origin of the frame of reference is

$$\tilde{\xi}_{\mu\nu} = (C_{\mu\nu}/r)e^{-(2\Lambda_s)^{-1/2}r} \qquad (r \neq 0) \tag{5.21}$$

($C_{00} - C_{11} - C_{22} - C_{33} = 0$), which is a Yukawa field of Compton wavelength $\lambda_C = (2\Lambda)^{-1/2}$, i.e., its range is $\sim L_0$, corresponding to a spin-2 particle of mass $\sim M_0 = 1.22$ GeV. The gravitational field is

$$\xi_{\mu\nu} = (C_{\mu\nu}/r)e^{-(2\Lambda_s)^{-1/2}r} + \tfrac{1}{4}\xi\eta_{\mu\nu} \tag{5.22}$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + \xi_{\mu\nu}$$

and ξ is the solution of

$$(\nabla^2 + \tfrac{4}{3}\Lambda_s)\xi = 0$$

i.e.,

$$\xi = (1/r)\{A\cos[(\tfrac{4}{3}\Lambda_s)^{1/2}r] + B\sin[(\tfrac{4}{3}\Lambda_s)^{1/2}r]\} \tag{5.23}$$

The Yukawa form of the gravitational potential of a quantum particle can be understood as arising from the screening of its gravitational charge due to polarization of the vacuum into uniton–antiuniton pairs, just as the electrostatic potential of a test charge in a metal has not the Coulomb but the Yukawa form as a result of its disturbing the equilibrium concentration of the Fermi electron gas. It represents a screened Newtonian potential and the screening length is just the Planck distance. At large distances ($r \gg L_0$), the dominant Newtonian potential is subject to Planck oscillations of wavelength $\lambda = 2\pi(\frac{3}{4}\Lambda)^{1/2} \sim \pi 3^{1/2}L_0$. At shorter distances ($r \sim L_0$), the shortrange Yukawa potential is effective (see Figure 1). The weak-field approximation remains valid even down to hadron dimensions. For instance, ξ_{00} changes only from 10^{-3} to 10^{-2} for a decrease in distance from $5.7L_0 = 0.9$ F to $3.5L_0 = 0.5$ F for a particle of mass 1 GeV. So a Yukawa variation in the gravitational field of a hadron must be expected to persist right up to its Compton wavelength. Only the order of magnitude of the spin-2 meson (mediating "strong" gravity) rest mass is predictable, though if $\Lambda = 1/L_0{}^2$ exactly, then it is $2^{1/2}M_0 = 1.73$ GeV.

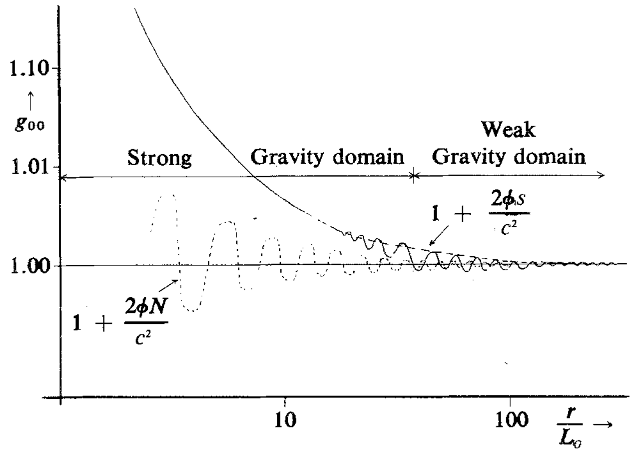

Fig. 1. Spatial variation in time component of hadron metric tensor due to Planck fluctuations. ϕ_s equals strong (Yukawa) gravitational potential. ϕ_N equals weak (Newtonian) gravitational potential.

6. CONCLUSION

The Einstein–Cartan theory leads to gravitational field equations containing a cosmological term that generates short-range massive gravitons possibly involved in strong interactions. Energy-nonconserving vacuum polarization on the Planck scale into uniton–antiuniton pairs gives the vacuum an intrinsic torsion and curvature, resulting in noncommutativity of translation operators and spontaneous breakdown of local Poincaré symmetry. The nonvanishing expectation value of the vacuum Bose sea spin contact interaction gives a cosmological term that is large compared with the energy-momentum tensor of matter only if the spin tensor is coupled to space-time torsion by the weak (Newtonian) gravitational constant. The assumption that it is due to the self-interaction of unitons created by "strong" rather than "weak" Planck fluctuations of the vacuum implies that hadron geometry markedly changes and differs from lepton geometry for distances approaching the particle Compton wavelength, which typically is of the order of magnitude of the "strong" Planck length. Vacuum polarization results in a modification of the Schwarzschild geometry of an unscreened classical gravitational monopole. This may be the underlying geometric basis for Yukawa's virtual fields, if we are willing to sacrifice the notion of an inert Euclidean geometry for the embedding space of hadrons and admit the existence of two gravitational coupling constants that divides matter into leptons and hadrons and whose disparity may have a cosmological origin (Sivaram and Sinha, 1976). Certainly the O'Raifeartaigh theorem would indicate that strong interactions do not have the space-time symmetry of the Poincaré group, and this is exactly what we would expect if they were a manifestation of local curved space. Implications for symmetry breaking, in particular how isospin and strangeness are related to the existence of gravitational dyons that are sources of the dual Riemann tensor, a tensor of $SL(2, C)^0$, as well as of the ordinary Riemann tensor, and the relation between SU_3 and spontaneously broken $SL(2, C) \otimes SL(2, C)^0$, will be discussed in a forthcoming paper.

APPENDIX A

In the Einstein–Cartan theory of gravitation, the generalization of a Riemann metric space with symmetric affine connection to a non-Riemann, nonmetric space with asymmetric affinity, the energy-momentum conservation law is

$$\nabla_\mu{}^+\Sigma^{\mu}_{\cdot\alpha} = \tau^{\sigma\cdot}_{\beta\nu} R^{\beta\nu}_{\cdot\sigma\alpha} \tag{A.1}$$

$\Sigma_{\mu\nu}$ is the canonical energy-momentum stress tensor, $R^{\beta\gamma}_{\cdot\sigma\alpha} = g^{\lambda\nu} R^{\beta}_{\lambda\sigma\alpha}$ and $\tau^{\sigma\cdot}_{\beta\nu} = g_{\nu\lambda}\tau^{\sigma\lambda}_{\beta\cdot}$, where $\tau^{\sigma\lambda}_{\beta\cdot}$ is the canonical spin angular momentum density which is related to the torsion tensor $\Omega^{\mu}_{\lambda\nu} = \frac{1}{2}(L^{\mu}_{\lambda\nu} - L^{\mu}_{\nu\lambda})$ and its contraction $\Omega_\lambda = \Omega^{\nu}_{\lambda\nu}$ by

$$\Omega^{\mu}_{\lambda\nu} + \delta_\lambda{}^\mu \Omega_\nu - \delta_\nu{}^\mu \Omega_\lambda = -\kappa\tau^{\mu}_{\lambda\nu} \tag{A.2}$$

The generalized divergence $\nabla_\mu{}^+\Sigma_\alpha{}^\mu$ is

$$\nabla_\mu{}^+\Sigma^{\mu}_{\cdot\alpha} \equiv (\nabla_\mu - 2\Omega_\mu)\Sigma^{\mu}_{\cdot\alpha} - 2\Omega^{\sigma}_{\mu\alpha}\Sigma^{\mu}_{\cdot\sigma} \tag{A.3}$$

From (A.2), putting $\mu = \nu$

$$2\Omega_\lambda = \kappa\tau_\lambda \tag{A.4}$$

so

$$\Omega^{\mu}_{\lambda\nu} = -\kappa(\tau^{\mu}_{\lambda\nu} + \tfrac{1}{2}\delta_\lambda{}^\mu\tau_\nu - \tfrac{1}{2}\delta_\nu{}^\mu\tau_\lambda) \tag{A.5}$$

Noting

$$\tau^{\sigma\cdot}_{\beta\nu} R^{\beta\nu}_{\cdot\sigma\alpha} = \tau^{\sigma\lambda}_{\beta\cdot} R^{\beta}_{\lambda\sigma\alpha} \tag{A.6}$$

then (A.1) becomes

$$\nabla_\mu \Sigma^{\mu}_{\cdot\alpha} = \tau^{\sigma\lambda}_{\beta\cdot} R^{\beta}_{\lambda\sigma\alpha} + 2\kappa(\tau_\mu\Sigma^{\mu}_{\cdot\alpha} - \tau^{\lambda}_{\mu\alpha}\Sigma^{\mu}_{\cdot\lambda} - \tfrac{1}{2}\tau_\alpha\Sigma) \tag{A.7}$$

where $\Sigma \equiv \Sigma^{\alpha}_{\cdot\alpha}$.

APPENDIX B

In Riemann–Cartan space, the generalized Bianchi identity is

$$\nabla_{[\mu} R^{\nu}_{\lambda\alpha\beta]} = 2\Omega^{\sigma}_{[\mu\alpha} R^{\nu}_{\lambda\sigma\beta]} \tag{B.1}$$

Contracting over ν, β

$$\nabla_\mu R_{\lambda\alpha} - \nabla_\beta R^{\beta}_{\lambda\alpha\mu} = 2(\Omega^{\sigma}_{\mu\alpha} R_{\lambda\sigma} - \Omega^{\sigma}_{\beta\alpha} R^{\beta}_{\lambda\sigma\mu}) \tag{B.2}$$

Taking the inner product of (B.2) with $g^{\lambda\alpha}$ and using the metric condition

$$\nabla_\mu g^{\lambda\nu} = 0 \tag{B.3}$$

and the antisymmetry property $R_{\lambda\sigma\alpha\mu} = -R_{\sigma\lambda\alpha\mu}$ of the curvature tensor, then

$$\nabla_\mu R = 2g^{\lambda\alpha}\Omega^{\sigma}_{\alpha\beta} R^{\beta}_{\lambda\sigma\mu} \tag{B.4}$$

which, with (A.5), gives

$$\nabla_\mu R = \partial_\mu R = 2\kappa(\tau^{\sigma\lambda}_{\beta\cdot} R^{\beta}_{\lambda\sigma\mu} - \tau^{\lambda} R_{\lambda\mu}) \tag{B.5}$$

The Einstein–Cartan field equations are

$$G^{\alpha}_{\cdot\beta} \equiv R^{\alpha}_{\cdot\beta} - \tfrac{1}{2}\delta_\beta{}^\alpha R = -\kappa\Sigma^{\alpha}_{\cdot\beta} \tag{B.6}$$

so

$$R = -\kappa\Sigma \tag{B.7}$$

and

$$R_{\lambda\mu} = -\kappa(\Sigma_{\lambda\mu} - \tfrac{1}{2}g_{\lambda\mu}\Sigma) \tag{B.8}$$

(B.5) finally becomes

$$\partial_\mu\Sigma = 2[\tau^{\sigma\lambda}_{\alpha\cdot} R^{\alpha}_{\lambda\sigma\mu} + \kappa(\tau_\lambda\Sigma^{\lambda}_{\cdot\mu} - \tfrac{1}{2}\tau_\mu\Sigma)] \tag{B.9}$$

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